Seminar on Six-functors formalisms

Wednesday 10 − 12, Room M 103 and online via this zoom link

The goal of the seminar is to give an introduction to the concept of 6 operations, following Mann [Man22] and Scholze [Sch23], as well as the ground work of Liu and Zheng [LZ15, LZ17]. We will essentially follow Scholze's lecture notes.

Talk 1 − Introduction. [Denis-Charles Cisinski, 18-10-2023]
We will give a quick overview of the example of sheaves on a locally compact Hausdorff topological space and Verdier duality and give an overview of the concept of 6 functor formalism. We will distribute the talks that are not assigned yet.

Talk 2 − Symmetric monoidal ∞-categories. [25-10-2023, Jana Nickel] Follow [Sch23, Lecture III].

Talk 3 − Construction of Six-functors formalisms. [8-11-2023, Vova Sosnilo] Follow [Sch23, Lecture IV].

Talk 4 − Poincaré duality. [15-11-2023, Johannes Gloßner] Follow [Sch23, Lecture V].

Talk 5 − Twisted ambidexterity and cohomological descent. [22-11-2023, Luca Pol] Follow [Sch23, Lecture VI and its appendices]. Explain the link between f-smoothness and the concept of universal local acyclicity.

Talk 6 − Topological spaces. [29-11-2023, Divya Ghanshani] Follow [Sch23, Lecture VII] and mention the link with Lurie's version of Verdier duality, following [Vol21].

Talk 7 − Étale sheaves. [6-12-2023, Jeroen Hekking] Follow [Sch23, Appendix to Lecture VII] (expanding a bit on relative purity and deformation to the normal cone).

Talk 8 − Coherent sheaves. [13-12-2023, Niko Naumann] Follow [Sch23, Lecture VIII]

Talk 9 − D-modules. [20-12-2023, Sebastian Wolf] Follow [Sch23, Appendix to Lecture VIII]

Talk 10 − Motivic sheaves. [10-01-2024, Marc Hoyois] Introduce Voevodsky's axiomatization of the six operations as done in [Kha21]. Gallauer's notes [Gal21] might be helpful as well. Explain the obvious connection with the previous talks.

Talk 11 − Universal property of motivic sheaves. [17-01-2024, Marco Giustetto] Explain the universal property following Drew and Gallauer [DG22].

Talk 12 − Rigidity Theorems. [24-01-2024, Giacomo Bertizzolo] Explain the comparison between étale sheaves and motivic sheaves with torsion coefficients, following Bachmann [Bac21b, Bac21a].

Talk 13 − Passage to stacks. [31-01-2024, Pier Federico Pacchiarotti] Follow [Sch23, Appendix to Lecture IV]. Discuss concrete computations in the $\mathbf{A}^1$-invariant setting following [KR22, Section 5].

Talk 14 − Nuclear sheaves. [7-02-2024, Niklas Kipp]

References

[Bac21a] Tom Bachmann. Remarks on étale motivic stable homotopy theory. with an appendix by Tom Bachmann and Marc Hoyois; arXiv:2104.06002, 2021.

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[DG22] Brad Drew and Martin Gallauer. The universal six-functor formalism. *Ann. K-Th. 7 (2022) 599-649*, 7:599–649, 2022. arXiv:2009.13610.

[Gal21] Martin Gallauer. An introduction to six-functor formalisms. arXiv:2112.10456, 2021.

[Kha21] Adeel A. Khan. Voevodsky's criterion for constructible categories of coefficients. available at `https://www.preschema.com/papers/six.pdf`, 2021.

[KR21] Adeel A. Khan and Charanya Ravi. Generalized cohomology theories for algebraic stacks. arXiv:2106.15001, 2021.

[KR22] Adeel A. Khan and Charanya Ravi. Equivariant generalized cohomology via stacks. arXiv:2209.07801, 2022.

[LZ15] Yifeng Liu and Weizhe Zheng. Gluing restricted nerves of ∞-categories. arXiv:1211.5294v4, 2015.

[LZ17] Yifeng Liu and Weizhe Zheng. Enhanced six operations and base change theorem for higher Artin stacks. arXiv:1211.5948v3, 2017.

[Man22] Lucas Mann. A p-adic 6-functor formalism in rigid-analytic geometry. arXiv:2206.02022, 2022.

[Sch23] Peter Scholze. Six-functor formalisms. Lecture notes for course WS 22/23, available at `https://people.mpim-bonn.mpg.de/scholze/SixFunctors.pdf`, 2023.

[Vol21] Marco Volpe. The six operations in topology. arXiv:2110.10212, 2021.