

Seminar on o-minimal structures Program

22.10.25 - Basic Definitions (Lyne Moser)

We will follow first [vdD, Chap. 1]: Definition of a structure (2.1), Lemma 2.1 and 2.2; Definition of an o-minimal structure (3.2), Lemmata 3.3, 3.4, 3.6. We will then complete with some basic properties of definable connected components (Prop. 2.18 in [vdD, Chap. 3]) and on definable families (Prop. 3.5 and Cor. 3.7 in [vdD, Chap. 3]).

29.10.25 - Examples (Lei Kuang)

Following [vdD, Chap. 1], we will see how to get o-minimal structures from model theoretic structures (Section 5) and then see the examples of the simplest o-minimal structure (Section 6), as well as the one of semi-linear sets (Section 7).

05.11.25 - Semi-algebraic sets (Wenjun Huang)

This is about real algebraic geometry, following [vdD, Chap. 2]. We will prove Thom's lemma (on the good behaviour of closure). See also Thm. 2.7 as well as Cor. 2.9 and 2.11. We will see also Noether's normalisation lemma (as well as Lemma 3.2 and Cor. 3.6).

12.11.25 - Monotonicity theorem (Zhenming Xu)

We will follow [vdD, Chap. 3], proving the monotonicity theorem (the fact that any definable map on an interval is piecewise constant or strictly monotone). We will also prove Cor. 1 and 2 as well as the finiteness lemma.

19.11.25 - Cell decomposition theorem (Kristina Dengler)

Still following [vdD, Chap. 3], we will prove the cell decomposition theorem (2.11).

26.11.25 - Dimension theory (Christoph Winges)

Follow [vdD, Chap. 4, §1].

03.12.25 - Euler characteristic (Bin Wu)

Follow [vdD, Chap. 4, §2].

10.12.25 - Curve selection (Ningyi Li)

We will follow [vdD, Chap. 6, §1]. discuss definable choice (Prop. 1.2) and prove the curve selection lemma (Cor. 1.5), which control closure of definable subsets through deformations along infinitesimal definable curves. Discuss also Lemmata 1.7 and 1.9, Prop. 1.10, Cor. 1.11, 1.12 and 1.13, as well as Prop. 1.14.

17.12.25 - Proper maps (Sonya Ragothaman)

We will follow [vdD, Chap. 6]: §3 on paths and partition of unity, §4 on curves and proper maps.

07.01.26 - Triangulations 1 (Xuefei Li)

We will follow §1 in [vdD, Chap. 8]. Discuss definitions on simplexes and complexes as well as Lemma 1.10.

14.01.26 - Triangulations 2 (Shenguanlin Lin - Austin)

State and prove the triangulation theorem (Thm. 2.9 in [vdD, Chap. 8, §1]).

21.01.26 - Applications (Matteo Munefó)

Discuss the criterion of definable equivalences through dimension and Euler characteristic (2.11 in [vdD, Chap. 8]) as well as the number of definable homeomorphism types (2.12 in [vdD, Chap. 8]). Explain the existence of definable continuous extensions (Prop. 3.3, Cor. 3.9 and 3.10 in [vdD, Chap. 8]).

28.01.26 - Definable quotients (Tashi Walde)

Discuss the existence of definable quotients [vdD, Chap. 10, §2]. This is another (non-trivial) application of triangulations in o-minimal geometry!

04.02.26 - Generic Fibers for Definable Families 1 (Jeroen Hekking/Lyne Moser)

This is the ground for the theory of sheaves in o-minimal geometry. We will follow now [Coste, Chap.5]. In this first part, we will discuss the sections on the space of ultrafilters of definable sets, on the o-minimal structure associated to an ultrafilter, and on the extension of definable sets.

11.02.26 - Generic Fibers for Definable Families 2 (Jeroen Hekking/Lyne Moser)

This talk is about bundles. We will discuss definable families as well as fiberwise and global properties of definable maps following [Coste, Chap.5].

References

- [vdD] L. van den Dries, *Tame Topology and O-minimal Structures*, London Math. Soc. Lecture Note 248. Cambridge Univ. Press, 1998 (pdf copy available [here](#))
- [Coste] Michel Coste, *An introduction to o-minimal geometry*, 1999 (lecture notes available [here](#))