Seminar on de Jong's alteration theorem
Program

21.10.22 - Introduction (Denis-Charles Cisinski)

28.10.22 - Preparation: semi-stable curves, (quasi)-excellent schemes (Robin Carlier)

Follow [Conrad, §2].

04.11.22 - First reductions (Sebastian Wolf)

Follow [Conrad, §3]

11.11.22 - Construction of good relative curves 1 (Johannes Gloßner)

In the first half of the talk, follow [Conrad, §4] on the construction of good curves fibrations. In particular, explain carefully the statement of Lemma 4.1, that will be the main goal of this talk and of the next one. The second half of the talk consists of stating and proving [Conrad, Prop. 5.2], one of the main ingredients being a Bertini theorem.

18.11.22 - Construction of good relative curves 2 (Andrea Panontin)

In this talk, we will prove [Conrad, Lemma 5.9] and complete the proof of [Conrad, Lemma 4.1].

25.11.22 - The three-point lemma (Massimo Pippi)

Follow [Conrad, §6].

02.12.22 - Hilbert schemes (Niklas Kipp)

Define Hilbert and Quot functors [Nitsure, 5.1.3], and recall the stratification by Hilbert polynomials. State and explain Grothendieck's semi-continuity Theorem [Nitsure, Thm. 5.10]. Explain Castelnuovo-Mumford regularity: definition of m-regular coherent sheaves on a projective space, and Mumford's theorem [Nitsure, Mumford] establishing the m-regularity of subsheaves of free sheaves of finite type (without proof). Sketch a proof of representability of Quot schemes [Nitsure, Theorem 5.15], following the steps [Nitsure, 5.5.3, 5.5.4, 5.5.5, 5.5.6, 5.5.7]. Discuss the quasi-projective version [Nitsure, Thm. 5.20]. Explain representability of the sheaf of morphisms between two projective schemes as an open subscheme of a Hilbert scheme.

09.12.22 - Stable curves (Ritheesh-Krishna Thiruppathi)

Define stable curves [Conrad, 8.1] (see also [DM, Def. 1.1]). Give examples [Conrad, 8.2, 8.4, 8.5]. Prove that stable curves are local complete intersections. For this, we use a flatness argument to reduce to the case of a stable curve over a field [Liu, Chapter 6, §6.3, Cor. 3.24] and prove that a stable curve can be locally embedded in a surface, using [Liu, §10.3, Prop. 3.7 (d)] (or use [Liu, Chapter 7, Prop. 5.15]). Recall the definition of the dualizing sheaf [Liu, Chapter 6, Def. 4.18] and recall the description of the dualizing sheaf for l.c.i. morphisms (as in [Liu, Chapter 6, Exercise 4.11]). Recall the case of a finite morphism as well [Liu, Chapter 6, Prop. 4.25]. Explain Theorem 1.2 and its corollary in [DM, pages 77–78].

16.12.22 - Deformations of stable curves (Pavel Sechin)

In first part, we will see all definitions needed to understand the statement of [Vistoli, Theorems 5.3 and 5.4]. Define formal deformations and their isomorphisms [Vistoli, 6.1 and 6.2], as well as (uni)versal deformations [Vistoli, 7.1 and 7.2] and state [Vistoli, Theorem 7.9] and its corollaries [Vistoli, Theorem 7.10 and 7.11]. Explain why any stable curve of genius g can be embedded in the projective space $\mathbf{P}^{5g-6}$ [DM, p. 78]. Explain Lemmata [DM, 1.3 and 1.4]. Discuss [DM, Prop. 1.5] and derive its corollaries [DM, 1.7 and 1.9].

23.12.22 - The stack of stable curves (Suraj Yadav)

Follow Conrad's exposition [Conrad, §9, pp. 51–63].

13.01.23 - Application of moduli stacks 1 (Benedikt Preis)

20.01.23 - Application of moduli stacks 2 (Suraj Yadav)

27.01.23 - Semi-stable curves over a DVR

03.02.23 - Resolving semi-stable curves over a regular base

10.02.23 - Controling the étale locus of the alteration (Niklas Kipp)

References

[BS] B. Bhatt & A. Snowden, *Refined alterations*, `https://www.math.ias.edu/~bhatt/math/alterationsepsilon.pdf`

[Conrad] B. Conrad, *Notes: alterations*. `http://math.stanford.edu/~conrad/249BW17Page/handouts/alterations-notes.pdf`

[de Jong] A. J. de Jong, *Smoothness, semi-stability and alterations*, Publ. Math. IHÉS, Vol. 83 (1996), pp. 51–93

[DM] P. Deligne & D. Mumford, *The irreducibility of the space of curves of given genus*, Publ. Math. IHÉS, Vol. 36 (1969), pp. 75–109

[Liu] Q. Liu, *Algebraic geometry and arithmetic curves*, Oxford Graduate Texts in Mathematics No. 6., Oxford University Press, 2002

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[Vistoli] Angelo Vistoli, *The deformation theory of local complete intersections*, arXiv:alg-geom/9703008, (1999).